Year 12 Mathematics Specialist 2019

Test Number 1: Complex Numbers

Resource Free

Name: _____ **Teacher: Mrs Da Cruz**

Marks: **20**

Time Allowed: **20 minutes**

Instructions: You **ARE NOT** permitted any notes or calculator. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

Question 1 **[2 marks]**

Express $z = -1 - \sqrt{3}i$ in polar form.

Question 2 **[1, 4 = 5 marks]**

Consider $f(z) = 2z^3 - 5z^2 + 22z - 10,\ z \in \mathbb{C}$.

a) Show that $f(0.5) = 0$.

b) Find all values, real and complex, for which $f(z) = 0$.

Question 3 **[3 marks]**

A quadratic equation with real coefficients has one of its roots as $z = 7 - 2i$. Find the equation.

Question 4 **[3 marks]**

If $z = \frac{1}{2-3i}$ then express z in cartesian form. Hence find $z.\bar{z}$

Question 5 **[5 marks]**

Using de Moivre's Theorem, find the exact value of $(1+i)^5 - (1-i)^5$

Question 6 **[2 marks]**

Complete this proof that de Moivre's Theorem hold for $n = -1$. *Ensure you provide your reasons when you use various rules (or show use of them).*

RTP: $(cis\ \theta)^{-1} = cis\ (-\theta)$

Proof: LHS = $(cis\ \theta)^{-1}$

= $\frac{1}{\cos\theta + i\sin\theta}$

= cis $(-\theta)$ = RHS

QED

Year 12 Mathematics Specialist 2019

Test Number 1: Complex Numbers

Resource Rich

Name: _____ **Teacher: Mrs Da Cruz**

Marks: **25**

Time Allowed: **25 minutes**

Instructions: You are permitted 1 A4 page of notes and calculators. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

Question 7 **[1, 1, 2, 2 = 6 marks]**

Let two complex numbers be $z = a(\cos\theta + i\sin\theta)$ and $w = b(\cos\phi + i\sin\phi)$. Determine the following in terms of a, b, ϕ and θ.

a) $Arg(\sqrt{w})$.

b) $\frac{|w|}{|z^2|}$.

c) $Arg\left(\frac{zw}{i}\right)$.

d) $|z^{-1}w^2|$.

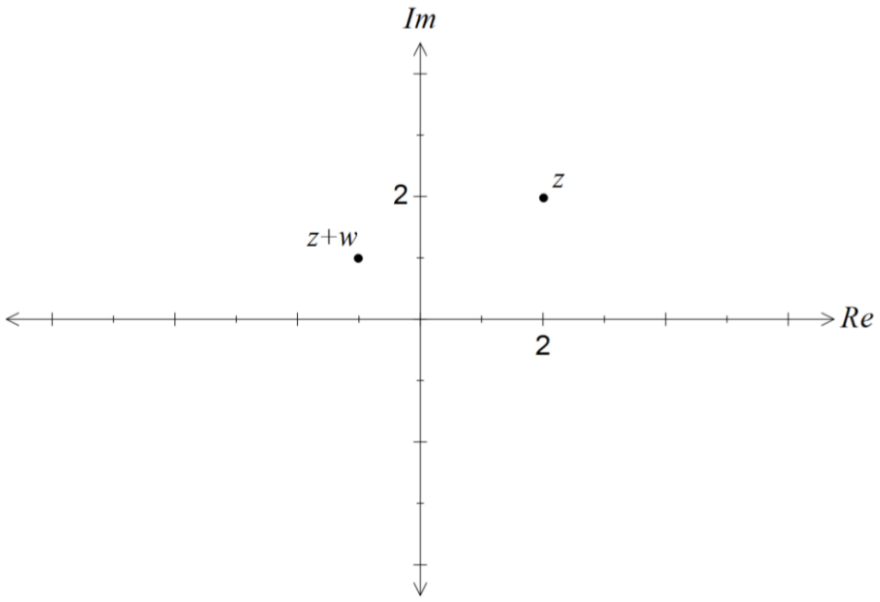
Question 8 **[1, 1, 2 = 4 marks]**

The complex numbers z and $z + w$ are shown on the Argand diagram below. On the same diagram plot and label the location of the following.

a) $i^2 z$.

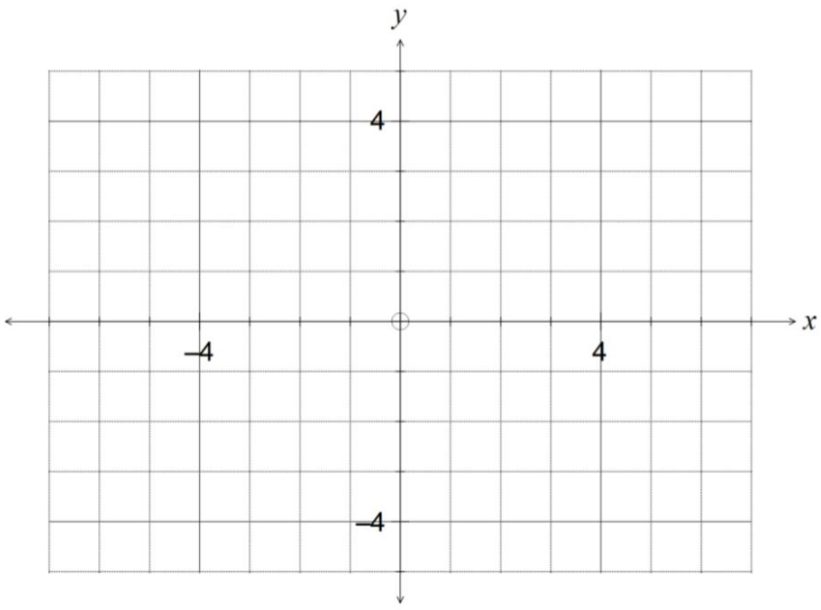
b) w.

c) $z^2 + wz$.



Question 9 **[4 marks]**

Sketch the region $|z| = 2|z - 3|$ in the complex plane.

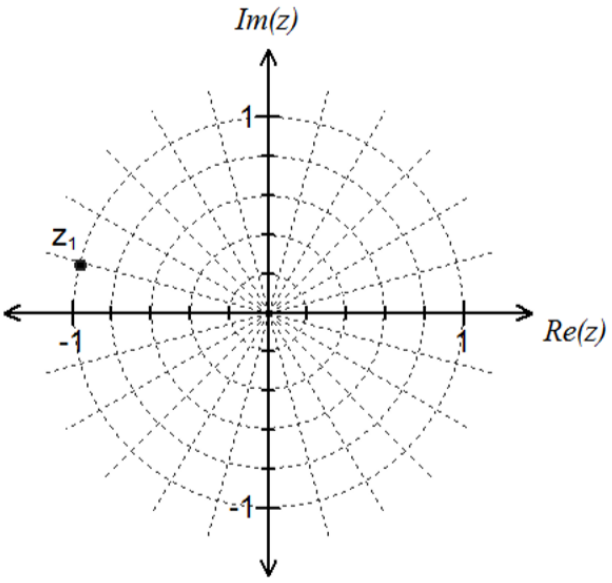


Question 10 **[4 marks]**

Solve the equation $z^4 = 2 + 2\sqrt{3}i,$ expressing all solutions in polar form.

Question 11 **[2, 2 = 4 marks]**

One solution to the equation $z^6 = a + bi$, where a and b are real constants, is shown on the diagram below.



(i) Plot all other solutions to the equation on the diagram. (2 marks)

(ii) Determine the values of a and b. (2 marks)

Question 12 **[3 marks]**

A sketch of the locus of a complex number z is shown below. Determine the maximum value for $\arg(z)$ correct to 0.01, where $0 \leq \arg(z) < 2\pi$.